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Modeling the thermal interaction of geothermal boreholes with aquifers using asymptotic expansion techniques

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Abstract. To design an optimally-sized geothermal heat exchanger it is crucial to accurately forecast its thermal response over the whole lifetime of the building. Since the presence of aquifers highly affects the heat exchange between geothermal boreholes and the ground, both heat transfer mechanisms, conduction and convection, must be taken into account in the ground. In the present work the existence of large disparities in time and length scales in the problem is exploited using asymptotic expansion techniques to develop a theoretical model capable of taking into account both aforementioned heat transfer mechanisms for any Peclet number of the groundwater flow.

1. Introduction

Energy plays a fundamental role in almost every challenge mankind faces nowadays. For instance, energy production represents around 60 % of total greenhouse gas emissions and indoor air pollution caused by fossil fuels used for household energy is responsible for 4.3 million deaths in 2012 [1]. Consequently, energy is explicitly addressed by the UN Sustainable Development Goals [1], for example, in goal number 7 that is focused on the improvement of energy efficiency in buildings. Taking into account that the heating and cooling of buildings represents more than 25% of the world energy consumption [2], any improvement in that front will significantly contribute to the achievement of the UN Sustainable Development Goals.

Nowadays, buildings are equipped with HVAC (heating, ventilation, and air conditioning) systems to satisfy their heating and cooling demands. One of the best renewable energy sources for HVAC systems is very low geothermal energy due to its widespread availability, its independence on local weather conditions, and its energy savings potential.

A geothermal HVAC system consists in a water-to-water heat pump connected to a geothermal heat exchanger composed of multiple vertical boreholes. Each borehole is equipped with one or more pipes wherein a liquid flows through and exchanges heat with the surrounding ground. Commonly, as a way of promoting heat exchange, the remaining space inside the borehole is filled up with grout of thermal conductivity k_b and thermal diffusivity α_b .

The correct sizing of a geothermal heat exchanger is crucial to ensure the successful harnessing of geothermal energy: a size too large means high initial costs and unreasonable payback times,



whereas satisfying the heating and cooling demands of a building with an undersized heat exchanger leads to more extreme temperatures in the heat carrying liquid and, consequently, to a drop in the overall efficiency of the HVAC system.

In order to design a geothermal heat exchanger with an optimal size it is indispensable to forecast its thermal response over the whole lifespan of the building, of typically 100 years. For that goal, the heat transfer problem in the boreholes and the ground must be solved. Unfortunately, a numerical simulation of the unsteady three-dimensional heat transfer problem of a real-world geothermal heat exchanger is nowadays beyond the reasonable computational cost. Therefore, simplified theoretical models must be developed and used instead, normally through their implementation in specialized simulation tools used for the analysis and design of geothermal heat exchangers.

Most theoretical models currently in use for the design of geothermal heat exchangers only take into account heat conduction in the ground [3, 4, 5, 6]. However, depending on the hydrogeological features of the ground it is likely to find flowing groundwater that fills up the voids within a geologic stratum [7, 8]. Depending on the location, the presence of such aquifers can highly affect the heat exchange between the boreholes and the ground [9]. This fact makes a relevant difference relying on how the terrain is used. In particular, a groundwater stream is beneficial when using the ground as a heat source/sink. In that case, only taking into account heat conduction leads to an oversized geothermal heat exchanger. On the contrary, if the ground is employed as seasonal storage, the presence of aquifers prevents recovering in wintertime the energy stored in summertime. Then, the pure conductive model provides an undersized geothermal heat exchanger.

So, to correctly size a geothermal heat exchanger it is crucial to account for the presence of groundwater streams when forecasting its thermal response, reason why the present ongoing work develops a theoretical model that incorporates both heat transfer mechanisms, conduction and convection, in the ground.

The article is organized as follows. In Section 2 a scale analysis of the problem is presented with the aim to know how to simplify it. Thereafter, the problem formulation including all performed assumptions is developed in Section 3. After that, in Section 4, the procedure followed to obtain the asymptotic solution to the problem is explained, while in Section 5 the results of a first case of study are shown. Finally, Section 6 is devoted to the conclusions.

2. Scale analysis of the problem

A typical geothermal borehole, like the one shown in Figure 1, presents depths H of the order of hundreds of meters and radii r_b of the order of tens of centimeters. Placed inside it are a series of pipes wherein a heat carrying liquid flows with a characteristic velocity V while it exchanges heat with the surrounding ground.

With the introduced length and velocity scales it is possible to construct three characteristic times, namely, the characteristic residence time $t_r \sim H/V$ of the fluid in the pipes, the characteristic transversal diffusion time $t_b \sim r_b^2/\alpha$, where α is the thermal diffusivity of the ground, and the characteristic longitudinal diffusion time $t_H \sim H^2/\alpha$. Computing these characteristic times using real-world values for H , r_b , V , and α reveals that t_r is of order minutes, t_b is of order hours, and t_H is of order centuries [10, 11].

Due to the heating and cooling needs of the building, the geothermal HVAC system imposes onto the boreholes a time varying heat injection rate that changes on an hourly, daily, weekly, monthly, and yearly basis. Hence, it presents a large spectrum of characteristic heat injection times t_q that goes from minutes up to decades. This work is focused on the most relevant operating conditions for which t_q is much larger than t_b and at the same time much smaller than t_H , leading to $t_r \ll t_b \ll t_q \ll t_H$ [10, 11].

Since the goal of this work is to consider the convection phenomenon, a fifth characteristic

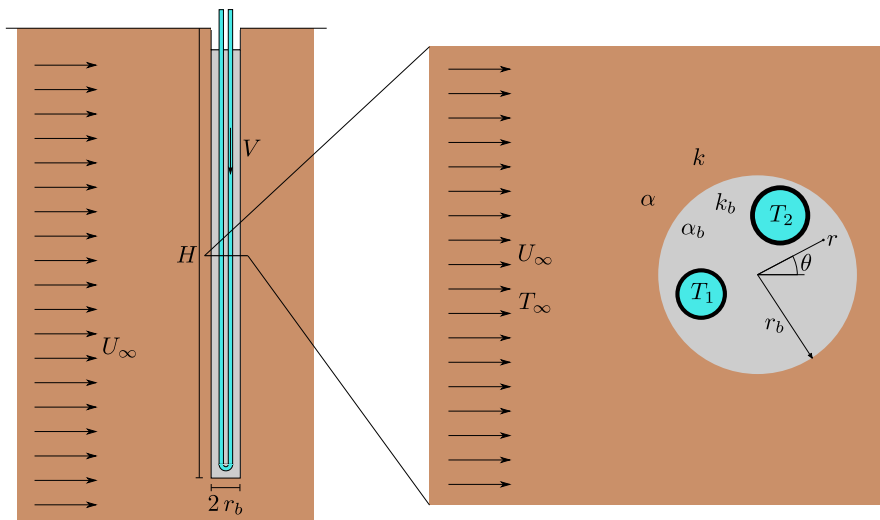


Figure 1. Sketch of a typical geothermal borehole, of depth H and radius r_b and with a single U-shaped probe inside, immersed in a groundwater stream of seepage velocity U_∞ and temperature T_∞ . Here, $T_1(t)$ and $T_2(t)$ are the temperatures of the fluid in each pipe of the probe, α_b and k_b are the thermal characteristics of the grout, and α and k the thermal characteristics of the ground.

time arises, namely, the characteristic residence time $t_c \sim r_b/U_\infty$ of the groundwater stream near the borehole, where U_∞ is the seepage velocity of the groundwater. The ratio between t_b and t_c is known as the Peclet number of the flow, $Pe = t_b/t_c$, and it is a nondimensional number that represents the relative importance of the convective terms over the diffusive terms in the energy conservation equation of the flow.

Most theoretical models for the thermal interaction between boreholes and aquifers assume the Peclet number to be small compared to unity [9, 12, 13, 14, 15, 16], leading to the following sequence of characteristic times: $t_r \ll t_b \ll t_c \sim t_q \ll t_H$. This assumption, however, is only valid for certain geographical locations since the Peclet number depends on geohydrological features of the ground such as its permeability, its thermal diffusivity, and the slope of the water table of the aquifer. Moreover, the Peclet number also depends on the borehole radius so that different installations in the same location lead to different Peclet numbers.

Figure 2 shows the Peclet number in terms of the permeability and the thermal diffusivity of different grounds [17] for a given water table slope of 5% and for two different radii that are typical for geothermal heat exchangers and for energy piles, respectively.

For vertical geothermal boreholes there are several types of grounds for which the hypothesis of small Peclet number fails, for example, for karst limestones, gravels, and coarse sands. If energy piles are considered instead, the case is even worse due to their larger radii, and the assumption of small Peclet number is only valid for fine sand and clay.

In summary, many real-world installations present Peclet numbers of order unity for which the existing theoretical models for the thermal interaction of aquifers with boreholes are not accurate. To overcome this problem the present work focuses on Peclet numbers of order unity, leading to the following sequence of characteristic times:

$$t_r \ll t_b \sim t_c \ll t_q \ll t_H. \quad (1)$$

The solutions corresponding to the limit of small Peclet numbers can still be recovered

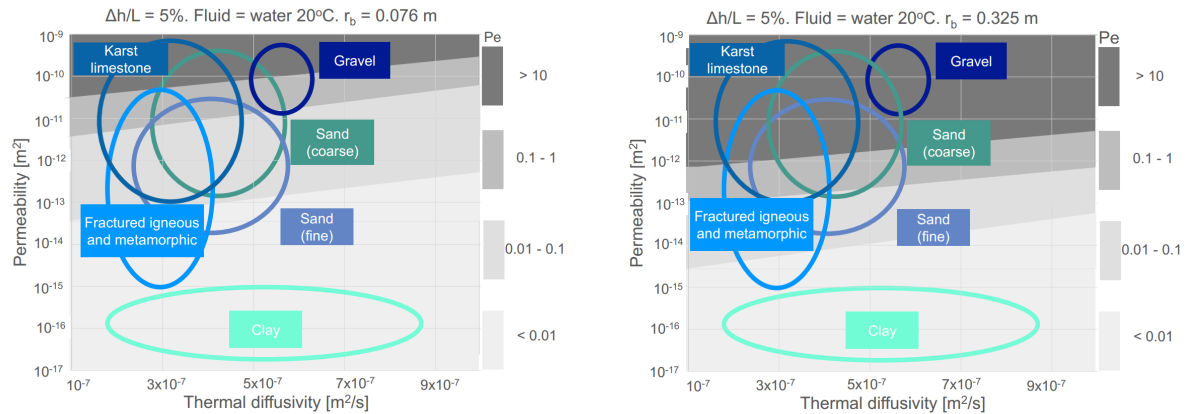


Figure 2. Peclet number for different kinds of grounds, for a water table slope of 5%, and for two different types of geothermal installations: (left) vertical borehole of radius $r_b = 0.076$ m and (right) energy pile of radius $r_b = 0.325$ m.

from the present work so that the theoretical model under development effectively represents a significant advancement compared to the state of the art.

3. Formulation of the problem

The just-presented scale analysis is going to be exploited next to formulate the simplified mathematical problem that represents the thermal interaction of an aquifer with a single borehole. Although, as mentioned above, a geothermal heat exchanger is composed of multiple boreholes, the present ongoing work focuses for the time being on the thermal response of one borehole only.

Since temperature differences in the problem are expected to be small, of the order of 10 - 20°C at most, their influence on the thermodynamic and transport properties of the groundwater is negligible allowing their values to be assumed constant. This simplification decouples the fluid-mechanical problem from the heat transfer problem, allowing the former one to be solved first.

Another important simplification results from the condition $t_q \ll t_H$ that, in first approximation, confines the problem to study to two-dimensional planes perpendicular to the borehole and only coupled to each other through the convective transport of heat along the pipes [10, 11]. Figure 1 shows one of those planes.

3.1. Fluid-mechanical problem

The groundwater movement through the small voids of the ground can be studied using porous media models [18] which substitute the governing Navier-Stokes equations by empirical laws. For the present case, in which the flow through the pores of the ground is dominated by viscous forces, Darcy's law is used for that [18].

The resulting model for the groundwater movement is analogous to the classical fluid-mechanical problem of the potential flow past a circular cylinder. Its solution is known analytically [19] so that the velocity field of the groundwater around the borehole is

$$v_r = U_\infty \left(1 - \frac{r_b^2}{r^2} \right) \cos \theta, \quad v_\theta = -U_\infty \left(1 + \frac{r_b^2}{r^2} \right) \sin \theta, \quad (2)$$

where (r, θ) are polar coordinates centered in the borehole, as shown in Figure 1.

3.2. Heat transfer problem

The fact that viscous forces govern the flow at pore scale implies that the thermal dispersion phenomenon is negligible [18] and that the assumption of local thermal equilibrium is valid [18]. All this allows the formulation of just one energy conservation equation for the groundwater-ground coupling with an effective thermal diffusivity α :

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} = \alpha \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right]. \quad (3)$$

For the time being, the ongoing work presented here focuses only on the temperature of the ground, and therefore a general heat flux $q(\theta, t)$ is imposed at the wall of the borehole, leading to the Neumann-type boundary condition

$$r = r_b : \quad -k \frac{\partial T}{\partial r} = q(\theta, t) \quad (4)$$

in which k is the effective thermal conductivity of the groundwater-ground coupling.

Finally, the unperturbed ground temperature T_∞ is imposed far from the borehole and as initial condition to the problem:

$$r \rightarrow \infty : \quad T(r, \theta, t) \rightarrow T_\infty, \quad t = 0 : \quad T(r, \theta, t) = T_\infty. \quad (5)$$

4. Limit of slowly varying heat injection rates

The heat transfer problem formulated in the previous section will be solved using asymptotic expansion techniques [20]. These mathematical methods exploit the presence of disparities in time and length scales to deliver approximate, but accurate, solutions to the problem. In the present case the fact that $t_b \ll t_q$ will be exploited which is known as the limit of slowly varying heat injection rates. This limit has already been studied extensively by the second author, although only for purely conducting grounds [10, 11, 21, 22, 23, 24, 25, 26].

4.1. Inner region

In this region, located at radial distances r comparable to the borehole radius r_b , the thermal inertia of the groundwater-ground coupling is negligible in the energy conservation equation formulated in (3). In consequence, the heat transfer in this region is governed by a quasi-steady convection-diffusion equation that can be solved using the separation of variables method [27], leading to a solution in terms of Mathieu functions [28, 29].

The integration constants are obtained from imposing the specified boundary conditions. However, only the one at the borehole wall, (4), can be imposed in the inner region as an outer region exists between the inner region and the boundary condition (5) far from the borehole.

4.2. Outer region

Contrarily to the inner region, in the outer region the thermal inertia is not negligible when compared to the convection and diffusion phenomena in the energy conservation equation of the groundwater-ground coupling given in (3). Consequently, an unsteady convection-diffusion equation needs to be solved. Fortunately, since the outer region is located far from the borehole, at radial distances of order $r \sim r_b(t_q/t_b) \gg r_b$, the velocity field is in first approximation not perturbed by the presence of the borehole, leading to a uniform stream given by

$$v_r = U_\infty \cos \theta + \mathcal{O} \left(\frac{t_b^2}{t_q^2} \right), \quad v_\theta = -U_\infty \sin \theta + \mathcal{O} \left(\frac{t_b^2}{t_q^2} \right). \quad (6)$$

The resulting differential equation can also be solved using the separation of variables method and the obtained solution is expressed in terms of modified Bessel functions [28].

Similarly to the inner region, the specified boundary conditions must be imposed to obtain the integration constants. But contrarily to the inner region, and because of its existence, the boundary condition (4) at the borehole wall can not be imposed now. Instead, in this region the boundary condition far from the borehole, (5), is satisfied.

4.3. Asymptotic matching

As a result of not satisfying all boundary conditions of the problem, the solutions obtained in the inner and outer regions present undetermined constants. These are obtained through asymptotic matching [20] which consists in requiring both solutions to be asymptotically equal at an intermediate region located at distances far from the inner region, but not as far as the outer region, so that the radial coordinate r satisfies $r_b \ll r \ll r_b(t_q/t_b)$.

Once the asymptotic matching is completed, the sought solution to the problem formulated in Section 3 is obtained. This solution is not exact, but as shown in the next section its accuracy is good enough for engineering purposes.

5. Results

To show the capabilities of the theoretical model under development, a comparison with a detailed numerical simulation obtained with the commercial program COMSOL [30] is presented next. The test case consists in a borehole of radius $r_b = 0.076$ m drilled into a ground with an effective thermal conductivity $k = 1$ W/(m K) and an effective thermal diffusivity $\alpha = 10^{-6}$ m²/s. The groundwater stream presents an unperturbed temperature of $T_\infty = 20^\circ\text{C}$ and its seepage velocity is $U_\infty = 2.3$ m/day leads to a Peclet number of $\text{Pe} = 2$.

At the borehole wall, a time-periodic heat flux with a period \mathcal{T} is imposed. Its expression can be expanded in Fourier series as follows:

$$q(\theta, t) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \hat{q}_n(\theta) e^{i\omega_n t}, \quad (7)$$

where $i = \sqrt{-1}$, \hat{q}_n is the n th harmonic, and $\omega_n = 2\pi n/\mathcal{T}$ is the corresponding angular frequency.

The time-periodic response of the ground to the heat flux imposed at the borehole wall can also be expanded in Fourier series, being \hat{T}_n the n th harmonic of the expansion:

$$T(r, \theta, t) = T_\infty + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \hat{T}_n(r, \theta) e^{i\omega_n t}. \quad (8)$$

Thanks to the linearity of the governing equations and boundary conditions, each harmonic \hat{T}_n can be obtained independently from the rest and it will only depend on the harmonic \hat{q}_n of the imposed heat flux at the borehole wall.

Figures 3 and 4 show the harmonic \hat{T}_n corresponding to an angular frequency of $\omega_n = 1.73 \cdot 10^{-5}$ s⁻¹ so that the ratio of characteristic times $t_b/t_q = (r_b^2/\alpha)/\omega_n^{-1}$ attains the rather large value of 0.1. The time-harmonic heat flux being imposed at the borehole wall is $\hat{q}_n(\theta) = (1 + \cos \theta + \sin \theta)100$ W/m² which leads to an asymmetric heat transfer problem in the ground.

Figure 3 focuses on the region close to the borehole to compare the asymptotic solution in the inner region, represented by black isolines, with the detailed solution obtained with COMSOL, represented by colored contours. The asymptotic model captures very well the real part of \hat{T}_n , while it fails to supply information on the imaginary part of \hat{T}_n (note the absence of black isolines

in the right plot of Figure 3). The imaginary part of \hat{T}_n , which is an order of magnitude or two smaller than the real part, can be captured by the asymptotic model by improving the solution to the inner region, a task already identified by the authors for the near future.

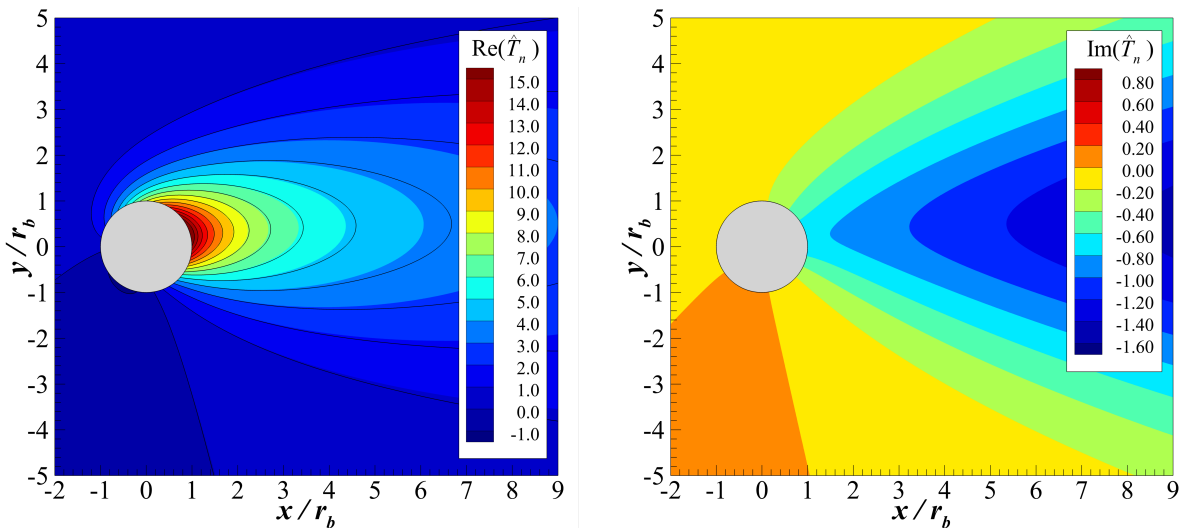


Figure 3. (Left) real and (right) imaginary part of the harmonic \hat{T}_n near the borehole obtained with COMSOL compared to the solution provided by the asymptotic model (solid isolines).

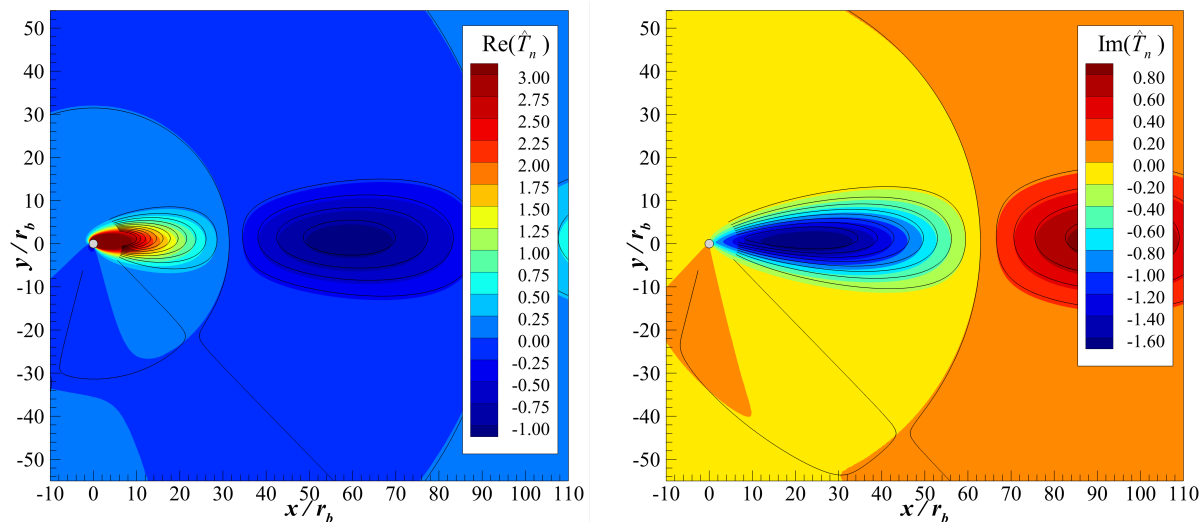


Figure 4. (Left) real and (right) imaginary part of the harmonic \hat{T}_n far from the borehole obtained with COMSOL compared to the solution provided by the asymptotic model (solid isolines).

Figure 4 also compares the asymptotic model with the solution provided by COMSOL, but this time at locations further away from the borehole. Hence, the solution to the outer region is being represented. The theoretical model is capable now of providing information about both real and imaginary parts of \hat{T}_n thanks to the presence of the thermal inertia in the energy conservation equation being solved in the outer region. The solution in the outer region also

reproduces very well the numerical solution provided by COMSOL for both parts, real and imaginary, of \hat{T}_n . Ways of further improving the outer solution have already been envisioned by the authors and will be analyzed in the near future.

6. Conclusions

Designing optimally-sized geothermal heat exchangers is crucial for the energy-efficient heating and cooling of buildings. To accomplish this it is necessary to accurately forecast the thermal response of the boreholes and their surrounding grounds over the whole lifespan of buildings.

When aquifers are present, the convective transport of heat they imply needs to be taken into account in the aforementioned forecast. As performing a detailed numerical integration of the unsteady three-dimensional heat transfer problem is computationally overwhelming, in this ongoing work a theoretical model for the thermal interaction of aquifers with geothermal boreholes is developed. The resulting model, derived by means of asymptotic expansion techniques, is valid for any Peclet number of the groundwater flow, thus significantly extending the applicability of existing theoretical models.

The comparison of the presented model with detailed numerical simulations reveals that its accuracy level is already acceptable for engineering purposes. Consequently, although the current model is still an ongoing work, it can already be employed to study the thermal response of one borehole. Moreover, it can also be used to study the thermal plume of that borehole and its impact on surrounding boreholes.

Acknowledgments

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